

NUMERACY RECOVERY: A PILOT SCHEME: EARLY INTERVENTION FOR YOUNG CHILDREN WITH NUMERACY DIFFICULTIES

by

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Abstract

A Numeracy Recovery scheme, funded by the Esmee Fairbairn Charitable Trust, is being piloted with 6-and 7-year-olds in six First Schools in Oxford. The scheme involves working with children who have been identified by their teachers as having problems with arithmetic. These children are assessed on eight components of early numeracy: (1) principles and procedures related to counting; (2) use of written arithmetical symbolism; (3) use of place value in arithmetic; (4) understanding and solution of word problems; (5) translation between concrete, verbal and numerical formats; (6) use of derived fact strategies for calculation; (7) arithmetical estimation and (8) memory for number facts. The children then receive weekly individual intervention in the particular components with which they have been found to have difficulty. To assess effects, children receive the WISC Arithmetic subtest the BAS Basic Number Skills subtest, and the WOND Numerical Operations test before the intervention begins, and at intervals of about 6 months. For the first 71 children in the project, Wilcoxon tests showed a significant improvement in standardized scores in all tests approximately a year following the start of intervention.

Numeracy recovery: a pilot scheme: Early intervention for young children with numeracy difficulties.

Specific difficulties in arithmetic have rarely received the same attention as specific difficulties in reading and writing; but it is likely that a higher proportion of adults experience persistent numeracy difficulties than by persistent literacy difficulties. Numerous studies demonstrate that a significant number of people experience difficulties with arithmetic, and for instance find it difficult to carry out arithmetical tasks of a type that are important in daily life, and have negative emotional reactions to arithmetic (Hitch, 1978; Cockcroft, 1982; ALBSU 1987; Cornelius 1992). There is evidence (ALBSU 1987) that a high proportion of adults with severe numeracy difficulties had already shown signs of arithmetical difficulties in the early school years. It should be made clear at the outset that arithmetic is by no means the only important part of mathematics, or the only part that creates difficulty for some individuals. It does, however, appear to create particular anxiety for many people.

In the area of reading skills, there has already been considerable emphasis on early identification of individual patterns of strengths and weaknesses, and their use in compensatory education for backward readers (e.g. Clay, 1985; Sylva and Hurry, 1995). However, there had been, especially until recently, little comparable work in the field of mathematical development and mathematics education.

Some previous research (Dowker, 1995, 1998) has suggested that specific arithmetical weaknesses are quite common; and are often restricted to particular components of arithmetic (e.g. written symbolism; memory for number facts; estimation). These findings have led to consideration of the desirability of a Numeracy Recovery scheme for assessing and ameliorating children's difficulties with specific areas of arithmetic in the early primary school years: before children have developed inappropriate arithmetical strategies based on fundamental misunderstandings, and/or developed negative or even phobic reactions to arithmetic.

While many forms of extra one-to-one teaching are likely to be helpful to children with difficulties, techniques that are targeted toward an individual child's specific weaknesses would seem likely to be most effective.

Current project

Such a Numeracy Recovery scheme is being piloted with 6-and 7-year-olds (mostly Year 2) in some First Schools in Oxford. It is currently being funded by the Esmee Fairbairn Charitable Trust. The scheme involves working with children who have been identified by their teachers as having problems with arithmetic. These children are assessed on eight components of early numeracy, which are summarized and described below. The children then receive weekly individual intervention (half an hour a week) in the particular components with which they have been found to have difficulty. The interventions are carried out by the classroom teachers, using techniques proposed by myself. The teachers are released (each teacher for half a day weekly) for the intervention, by the employment of supply teachers for classroom teaching. Each child remains in the program for 30 weeks, or until their teachers feel they no longer need intervention; whichever is shorter. New children join the project periodically.

The project has been set up in six Oxford primary schools. Some schools began intervention work in the autumn term of 1998; others were delayed in starting as a result of OFSTEDs and SATs, and resulting pressures on the teachers. However, the project is now well underway in all six schools. 78 children (about 15% of the children in the relevant classes) have so far begun or undergone intervention,

Components that are the focus of the project

The components addressed here are not to be regarded as an all-inclusive list of components of arithmetic, either from a mathematical or educational point of view. Rather, a few components have been selected because discussion with teachers has indicated to be important in early arithmetical development, and because research (Dowker, 1995; Dowker, 1998) has shown to vary considerably between individual children in the early years.

The components that are the focus of the project include (1) principles and procedures related to basic counting; (2) use of written arithmetical symbolism; (3) use of place value in arithmetic; (4) understanding and solution of word problems; (5) translation between concrete, verbal and numerical formats; (6) use of derived fact strategies for calculation; (7) arithmetical estimation and (8) memory for number facts.

Remediation for these components principally involves techniques devised by the author, supplemented by exercises and games taken from published materials (e.g. Burgess, 1995 a, b; Long, 1996; Preston, 1998; Scarry, 1998; Straker, 1996). Techniques devised and used by the teachers themselves also play a major role in the project.

The components, and the main intervention techniques, will now be summarized:

(1) Principles and procedures related to counting:

This component includes accurate counting of sets of objects; rote verbal counting; understanding of some of the principles involved in counting (Greeno, Riley and Gelman, 1984); and repeated addition and subtraction by 1. Very basic counting rarely presents problems for 6-year-olds, even those who are weak at arithmetic. The areas that are most likely to present difficulties for children in this age group are:

- (a) Understanding of the order-irrelevance principle. In other words, the child needs to be able to understand that the result of counting a set of items will not change if the items are counted in a different order, whereas adding or subtracting an item will change the number.
- (b) Repeated addition by 1, where children counts a set of 8 counters, and are asked how many there will be if one more counter is added. This is repeated up to 20.
- (c) Repeated subtraction by 1, where children count a set of 10 counters, and are asked how many there will be if one is taken away. This is repeated down to zero.

Intervention:

For the order irrelevance principle, children practice counting and answering cardinality and order-irrelevance questions about very small numbers of counters (up to 4); and are then given further practice with increasingly large sets. For repeated addition by 1 and repeated subtraction by 1, children are given practice in observing and predicting the results of such repeated additions and subtractions with counters (up to 20). They will then be given verbal 'number after' and 'number before' problems: "What is the number before 8?", "What is the number after 14?", etc.

(2) Written symbolism for numbers

There is much evidence that children often experience difficulties with written arithmetical symbolism of all sorts, and in particular with representing quantities as numerals (Ginsburg, 1989; Fuson, 1992). With regard to this component, children are asked to read aloud a set of single-digit and two-digit numbers. A similar set of numbers is dictated to them for writing.

Intervention: Children practice reading and writing numbers. Children with difficulties in reading or writing two-digit numbers (tens and units) are given practice in sorting objects into groups of ten, and recording them as "20", "30", etc. They will then be given such sorting and recording tasks where there are extra units as well as the groups of ten.

(3) Understanding the role of place value in number operations and arithmetic

This involves the ability to add 10s to units ($20 + 3 = 23$); the ability to add 10s to 10s ($20 + 30 = 50$); and the ability to combine the two into one operation ($20 + 33 = 53$). A related task involves pointing to the larger number in pairs of 2-digit numbers, that vary either just with regard to the units (e.g. 23 versus 26); just with regard to the 10s (e.g. 41 versus 51); or where both tens and units vary in conflicting directions (e.g. 27 versus 31; 52 versus 48).

Intervention:

Children are shown the addition of tens to units and the addition of tens to tens in several different forms:

- (i) Written numerals;
- (ii) Number line or number block;
- (iii) Hands and fingers in pictures;
- (iv) 10-pence pieces and pennies;
- (v) Any apparatus (e.g. Multilink or Unifix) with which the child is familiar. The fact that these give the same answers should be emphasized. Children whose difficulties are more specific to the use of place value in arithmetic may be given practice with arithmetical patterns such as: " $20 + 10$; $20 + 11$; $20 + 12$ ", etc; being encouraged to use apparatus when necessary.

(4) Word problem solving

This component involves comprehending addition and subtraction story problems of various semantic types (see DeCorte and Verschaffel, 1987); selecting the appropriate operations; and solving the problems.

Intervention

Children are given addition and subtraction word problems, which are discussed with them:

"What are the numbers that we have to work with?" "What do we have to do with the numbers?" "Do you think that we have to do an adding sum or a taking-away sum?" "Do you think that John has more sweets or fewer sweets than he used to have?", etc. They are encouraged to use counters to represent the operations in the word problems, as well as writing the sums numerically.

(5) Translation between arithmetical problems presented in concrete, verbal and numerical formats

Translation between concrete, verbal and numerical formats has been suggested by several people to be a crucial area of difficulty in children's arithmetical development. For example, Hughes (1986) reported that many primary school children demonstrate difficulty in translating between concrete and numerical formats (in either direction), even when they are reasonably proficient at doing sums in either one of these formats and has suggested that this difficulty in translation may be an important hindrance to children's understanding of arithmetic.

The current project includes tasks of translating in all possible directions between numerical (written sums); concret (operations with counters); and verbal (word problem) formats for both addition and subtraction.

For example, in translating from verbal to numerical, they are presented with word problems, children are presented with word problems (e.g. 'Katie had five apples; she ate two, so now she has three left'), and are asked to "write down the sum that goes with the story"

Intervention

Children are shown the same problems in different forms, and shown that they give the same results. They are also encouraged to represent word problems and concrete problems by numerical sums, and to represent numerical problems and word problems by concrete objects.

(6) Derived fact strategies in addition and subtraction

One crucial aspect of arithmetical reasoning is the ability to derive and predict unknown arithmetical facts from known facts, for example by using arithmetical principles such as commutativity, associativity, the addition/ subtraction inverse principle, etc.

Children are given the Addition and Subtraction Principles Test developed by Dowker (1995, 1998). In this test, they are given the answer to a problem and then asked them to solve another problem that could be solved quickly by the appropriate use of an arithmetical principle (e.g. they may be shown the sum " $23 + 44 = 67$ " and then asked to do the sum $23 + 45$, or $44 + 23$). Problems preceded by answers to numerically unrelated problems are given as controls. The children are asked whether "the top sum" helps them to do "the bottom sum", and why. The actual addition and subtraction problems involved will vary in difficulty, ranging from those which the child can readily calculate mentally, through those just beyond the child's calculation capacity, to those very much too difficult for the child to solve. The particular derived fact strategies that are the main focus of this project are those involving commutativity (e.g. if $8 + 6 = 14$, then $6 + 8 = 14$); the associativity-based $N + 1$ principle (if $9 + 4 = 14$, then $9 + 5 = 14 + 1 = 15$) and the $n - 1$ principle (e.g. if $9 + 4 = 13$, then $9 + 3 = 13 - 1 = 12$).

Intervention:

Children are presented with pairs of arithmetic problems. The 'derived fact strategy' techniques are pointed out and explained to them; and they are invited to solve similar problem. If they fail to do so, the strategies are demonstrated to them for single-digit addition and subtraction problems, with the help of manipulable objects, and of a number line; and will again be invited to carry out other derived fact strategy problems.

(7) Arithmetical estimation

The ability to estimate an approximate answer to an arithmetic problem, and to evaluate the reasonableness of an arithmetical estimate, are important aspects of arithmetical understanding. In assessing and remediating this component, children are given a task devised by the author (Dowker, 1996). They are presented with a series of problems of varying degrees of difficulty, and with estimates made for these problems by imaginary characters (Tom and Mary). The children are asked (a) to evaluate "Tom and Mary"'s estimates on a five-point scale from "Very good" to "Very silly"; and (b) to suggest "good guesses" for these problems themselves. Once again, the actual addition and subtraction problems involved vary in difficulty, ranging where possible from those which the child can readily calculate mentally, through those just beyond the child's calculation capacity, to those very much too difficult for the child to solve.

Intervention:

Children are shown other arithmetical estimates by "Tom and Mary" and asked to evaluate them. They are encouraged to give reasons for their evaluations.

(8) Number fact retrieval

Although most psychologists, educators and mathematicians agree that memorization of facts is not the essence of arithmetic, knowledge of number facts does contribute to efficiency in calculation (cf. Merttens, 1996), and is a significant factor in distinguishing between mathematically normal and mathematically 'disabled' children (Russell and Ginsburg, 1982). In this study, this skill is principally assessed through Russell and Ginsburg's (1982) Number Facts Test.

Intervention:

Children are presented with some of the basic addition and subtraction facts (e.g. $3 + 3 = 6$; $6 + 6 = 12$). They are presented with the same sums repeatedly in the same session and in successive sessions. They also play 'number games' (e.g. some from X) that reinforce number fact knowledge.

Evaluation of effectiveness: some preliminary results

The children in the project, together with some of their classmates and children from other schools, are given three standardized arithmetic tests: the British Abilities Scales Basic Number Skills subtest (1995 revision), the WOND Numerical Operations test, and the WISC Arithmetic subtest. The first two greatest emphasis on computation abilities and the latter on arithmetical reasoning. The children are retested at intervals of approximately six months.

The project and its evaluation are still in relatively early stages. However, the initial, 6-month and 1-year standardized test scores of the first 71 children to take part in the project have now been analyzed. Not all of the data from 'control' children are yet available, but the tests are standardized, so it is possible to estimate the extent to which children are or are not improving relative to others of their age in the general population.

Results so far have been very promising. The median standard scores on the BAS Basic Number Skills subtest were 94 initially and 100 after one year. The median standard scores on the WOND Numerical Operations test were 88.5 initially and 91 after one year. The median standard scores on the WISC Arithmetic subtest were 6 initially, and 8 after one year. Wilcoxon tests showed that the improvements on the BAS Basic Number Skills subtest ($W = 589$) and WISC Arithmetic subtest ($W = 266$) were significant at the 0.01 level, and that the improvement on the WOND Numerical Operations subtest ($W = 794$) was significant at the 0.05 level.

The retesting and evaluation is intended to be carried out over a period of at least 3 years to assess whether gains made in the project are preserved in the longer term. A long-term goal is to compare the effects of this project with those of other intervention techniques, and of individual attention as such, so as to assess its specific effectiveness.

Teacher's comments:

The reactions of the teachers in the schools concerned have been very positive. They have expressed enthusiasm over the chance to work with children on an individual basis, and feel that the children are enjoying the project and are making considerable improvements. Some of them have said that involvement in the project is also giving them good ideas for general classroom arithmetic teaching.

A few of their comments may serve as an appropriate tentative conclusion at this stage:

"The children are responding very well to the materials and to the extra support... They are working through activities linked to basic number skills to establish and reinforce early concepts. Feedback from staff, children and parents has been very positive."

'(The project) has given us valuable information about pupils' learning needs in a core subject, and has provided us with the funding to support the most needy children with individual tuition... As a consequence, we have seen the targeted pupils improve considerably in competence and confidence.'

'Working with children individually gives greater opportunity for analysing their thinking through individual questioning... There is more time and opportunity for using apparatus and asking children to demonstrate what they are doing. These children are often very reluctant to verbalise what they are thinking and in a whole class or even small group situation, there is not the time to wait for or expect their replies. By giving the children 'thinking time', their confidence and willingness to 'have a go' develops as they offer explanations...The children seem to enjoy coming to the sessions and it has been possible to raise their self-esteem in mathematics in most cases.'

Several teachers considered that this individualized work complemented the classroom-based Numeracy Hour well, to provide a particularly effective combination.

General implications for transformation of learning

There are several implications for effective transformation of arithmetical (and possibly other) learning more generally:

- (1) Individualized work with children who are falling behind in arithmetic has a significant impact on their performance. The amount of time given to such individualized work does not, in many cases, need to be very large to be effective: these children received approximately half an hour a week, and showed considerable benefits.
- (2) Children are rarely uniformly weak at all aspects of arithmetic. It is misleading and potentially damaging to describe a child as globally 'good' or 'bad' at arithmetic. Although future research will be needed to compare the effectiveness of different types of program, it appears that diagnosis of, and intervention, in the specific areas of children's weaknesses is likely to prove particularly effective.
- (3) The study strongly supports the view that children's arithmetical difficulties are highly susceptible to intervention. It is not the case that a large number of children are simply 'bad at maths'. It is particularly notable that some of the greatest improvement occurred in the WISC Arithmetic subtest: a test sometimes regarded as a measure of predominantly 'innate' intelligence.
- (4) Collaborations between teachers and researchers are desirable in addressing educational problems.

References

- ALBSU (1992). Literacy, numeracy and adults: evidence from the National Child Development Study. London: Adult Literacy and Basic Skills Unit.
- BURGESS, L. (1995a). Counting: key skills in maths for ages 5 to 7. Oxford: Heinemann.
- BURGESS, L. (1995b). Pattern: key skills in maths for ages 5 to 7. Oxford: Heinemann.
- CLAY, M. M. (1985). The early detection of reading difficulties: a diagnostic survey with recovery procedures, 3rd edition. Auckland, N.Z.: Heinemann.
- COCKCROFT, W.H. (1982). Mathematics counts. London: HMSO.
- CORNELIUS, M. (1992). The numeracy needs of graduates in employment. University of Durham, unpublished manuscript.
- DECORTE, E. and VERSCHAFFEL, L. (1987). The effect of semantic structure on first-graders' strategies for solving addition and subtraction word problems. *Journal for Research in Mathematics Education*, 18, 363-381
- DOWKER, A.D. (1995). Children with specific calculation difficulties. *Links* 2, 2, 7-11.
- DOWKER, A. (1996). Arithmetical estimation: when does it begin? Poster presented at British Psychological Society Developmental Section Conference, Oxford, September 11th-12th, 1996.
- DOWKER, A. (1998). Individual differences in arithmetical development. In C. Donlan (ed.): *The development of mathematical thinking*. London: Taylor and Francis.
- FUSON, K. (1992). Research on learning and teaching addition and subtraction of whole numbers. In: G. Leinhardt, R. Putnam and R.A. Hattrop (eds.) *Analysis of arithmetic for mathematics teaching*. Hillsdale, N.J.: Erlbaum.
- GINSBURG, H.P. (1989). *Children's arithmetic*. New York, N.Y.: Van Nostrand.
- GREENO, T.G., RILEY, M.S. and GELMAN, R. (1984). Young children's counting and understanding of principles. *Cognitive Psychology*, 16, 94-143
- HITCH, G. (1978). The numerical ability of industrial apprentices. *Journal of Occupational Psychology*, 51, 163-176.
- HUGHES, M. (1986). *Children and number*. Oxford: Basil Blackwell.
- LONG, L. (1996). *Domino 1,2,3: a counting book*. London: Franklin Watts.
- PRESTON, R. (1998). *Number puzzles and sums*. Swindon: Early Learning Centre.
- RILEY, M.S., GREENO, J.G., and HELLER, J.I. (1983). Development of children's problem-solving ability in arithmetic. In H.P. Ginsburg (ed.) *The development of mathematical thinking*. New York: Academic Press.

RUSSELL, R. and GINSBURG, H.P. (1984). Cognitive analysis of children's mathematical difficulties. *Cognition and Instruction*, 1, 217-244.

SCARRY, R. (1998). *It's fun to learn sums*. London: Hamlyn.

STRAKER, A. (1996). *Mental maths for ages 5 to 7: teachers' book*. Cambridge: Cambridge University Press.

SYLVA, K. and HURRY, J. (1995). *Early intervention in children with reading difficulties*. London: School Curriculum and Assessment Authority Discussion Papers, No. 2.

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